Random phase encoding of composite fully complex filters

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The mapping of complex-valued functions onto phase-only spatial light modulators is examined. Random phase encoding effectively adds amplitude control to the phase-only filter and can be used to trade off systematic errors of the phase-only filter for random errors. This is illustrated for the problem of recognizing a three-dimensional object from arbitrary views. The complex-valued composite filters that constitute a filter bank design are encoded by phase-only and pseudorandom methods. The best recognition probabilities are achieved by blending the two methods so that only the smallest amplitudes are randomly encoded.

By properly composing a filter from several views of an object it is possible to recognize the object despite distortion, thus achieving a degree of distortion invariance. One general approach has been to form a filter bank that uses multiple composite filters. The design of linear phase coefficient composite filter banks produces a set of complex-valued filters.[1] Synthetic discriminant function[2] and minimum average correlation energy[3] filters are also complex valued. These single composite filters have since been generalized into the form of filter banks.[1,4] The hybrid composite (HC) filter bank combines the properties of all these filter banks through the selection of specific values for the two parameters α_1 and α_2.[4] This study uses a specific HC filter bank that is a balanced combination of synthetic discriminant function, minimum average correlation energy, and linear phase coefficient composite properties.[4] Therefore the results reported here are relevant to a variety of composite filters of current interest.

A significant and recognized problem limiting the use of composite filters in real-time optical correlators is that currently available filter plane spatial light modulators (SLM's) do not produce full-complex modulation.[5,6] This limitation is known to modify the peak intensities and produce false peaks through nonlinear intermodulation of the composite signals.[7] Casasent and Rozzi originally noted that peak fluctuations could dramatically change peak correlation intensities while keeping only the phase of the full complex design and that even a small degree of amplitude control greatly improves recognition.[5] Previous solutions to minimize the degradation caused by limited range SLM's required numerically intensive optimization techniques.[8,9] Faster encoding procedures are needed for those applications in which the time available for optimization is a limiting factor.

Pseudorandom encoding is a specific encoding technique for mapping full-complex filters onto phase-only SLM's.[10] It is a fast procedure because it requires only one function calculation or table look-up operation per pixel. The encoding procedure adds amplitude control to the phase-only filter[11] (POF) through the addition of phase offsets $\delta\psi_i$ that have specified statistical properties. For a uniform random distribution of spread ν_i the effective amplitude control achieved at the ith SLM pixel is known to be

$$\overline{a}_i = \langle \exp(j\delta\psi_i)\rangle = \mathrm{sinc}(\nu_i/2\pi)\,, \tag{1}$$

where $\langle\,\rangle$ is the expected value operator. In the encoding procedure the value of the amplitude in Eq. (1) is set to that of the desired full-complex modulation, and then Eq. (1) is inverted to yield the spread ν_i. For each pixel a randomly generated number is scaled by the appropriate spread to produce a phase offset $\delta\psi_i$ with the appropriate statistical properties. The random phase offsets are added to the phases ψ_i of the desired full-complex modulation to complete the encoding. Inasmuch as the values of Eq. (1) range between zero and one we always assume that the full-complex modulation is normalized so that its maximum amplitude is unity. Whereas no individual pixel actually produces amplitude modulation at the SLM plane, we have shown that the resulting far-field diffraction pattern is well approximated by treating each pixel as if it produced an average amplitude modulation $\overline{a}_i$.[10]

An indicator of quality of a pseudorandom encoded filter (PRF) is the diffraction efficiency (under uniform illumination)

$$\eta \equiv \frac{1}{N}\sum_{i=1}^{N} \overline{a}_i^{\,2}\,, \tag{2}$$

where N is the number of pixels of the SLM. The diffraction efficiency for the PRF represents the fraction of the energy illuminating the SLM that is used to form the diffraction pattern of the full-complex filter (FCF). In fact, it is exactly the diffraction efficiency of the desired, but unachievable, full-complex filter. The remaining $1-\eta$ fraction of the energy from the PRF is diffracted into a white-noise pattern resembling speckle. With the energy divided between desired signal and random noise it is clear that, as η increases

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toward unity, the encoded filter will be less noisy and will more closely approximate the full-complex filter. Our searching for optimal γ is much like Juday's optimization of gain parameter G.[12] For phase-only SLM's performance is independent of G, and the traditional POF[11] always results. There is, however, an optimal value of γ.

The traditional POF,[11] for which all amplitudes of the full-complex filter are mapped to unity, may also be viewed as another type of encoding. The POF can be considered to have unity diffraction efficiency. However, as noted in Ref. 7, the encoding also introduces systematic (rather than random) errors between the desired full-complex and realized phase-only filter.

The question considered in this study is whether the amplitude control offered by pseudorandom encoding can be used to improve performance of filter banks over that possible with the phase-only filters. We analyze this problem by designing a filter bank to recognize a specific object, encoding the complex filters to POF's and PRF's, and comparing the performance of the encoded filter banks at recognizing the object in the presence of noise, clutter, and distortion. The filters are designed for implementation on a $4f$ correlator containing a 32×32 pixel amplitude-only SLM in the input scene plane and a 64×64 pixel phase-only SLM in the filter plane.

The HC filter bank design follows identically the steps described in Ref. 4. This includes the identical choice of parameters $\alpha_1 = \alpha_2 = 0.4$. In the design presented here the goal is to identify the Space Shuttle and reject all other aircraft types. The training set consists of 36 images obtained from a three-dimensional Space Shuttle model that is viewed with an altitude angle of 60°, rotated uniformly in azimuth from 0° to 360° in 10° increments, and then projected to form 32×32 pixel silhouette (i.e., binary amplitude) images. Nontarget objects are not needed for obtaining clutter-resistant HC filters and thus were not used.[4] The training images are zero padded to 64×64 pixel images. The HC filters are derived from these training images in the form of impulse responses and then fast Fourier transformed to produce the frequency plane filters. These are the filters that are encoded by various methods.

Three nontarget aircraft have also been chosen to represent clutter objects for simulations of filter bank performance. Silhouette images of these objects are taken for the same view angles and approximate scale as the target object. Representative views of the Space Shuttle and of one clutter aircraft are shown in Fig. 1. The noise shown in the figure was added only for the specific set of tests described below. Performance of the filter banks is characterized by the minimum probability of error (MPE).[4] One achieves the MPE by setting the decision threshold to produce the least total number of false alarms and misses. In our simulations we calculate MPE empirically. First we find the value of the peak response of the filter bank for the in-class object and the maximum peak value of the 3 clutter objects for each of their 36 views. The MPE is then the minimum sum of false alarms and misses divided by 72 for all possible threshold settings. Although the filter bank design produces as many filters as training images (in this case 36), one can usually achieve adequate recognition by selecting a subset of the filters that have the largest discrimination-to-noise ratios.[4] For our simulations we calculated MPE for filter banks of 1–5 filters.

In a preliminary simulation we found that filter banks using POF's usually had fewer recognition errors than filter banks using fully encoded PRF's. This result is due to the low diffraction efficiency (only a few percent) of the FCF's, which consequently introduces too much random noise. This result led us to consider a blending of encoding procedures so that only some of the pixels are pseudorandom encoded and the rest are phase-only encoded. Because the lowest amplitudes of the FCF's produced the most systematic error for phase-only encoding, we now pseudorandom encode only those amplitudes that are below a given threshold. Currently the threshold that gives the smallest value of MPE is found empirically by repeated simulations. The amount of random encoding for a given amplitude threshold can be quantified in terms akin to those for diffraction efficiency [Eq. (1)] as

$$\eta_r = \frac{1}{N} \sum_{i=1}^{N_r} \overline{a}_i^{\,2}, \tag{3}$$

where N_r is the number of pixels below threshold that are random encoded. A relative measure of the amount of pseudorandom encoding is $\gamma = \eta_r/\eta$. A γ equal to zero corresponds to phase-only encoding, and a γ of unity corresponds to pseudorandom encoding all N pixels.

The results of the simulations of MPE for various encodings and two (related) sets of test imagery are summarized in Table 1. For each set of test imagery the table presents MPE for filter banks composed of FCF's, POF's, and PRF's. The first of the two PRF columns reports the lowest value of MPE found for all values of the encoding parameter γ. The corresponding encoding parameters ranged from 0.002 to 0.07. The second PRF column reports MPE for a single fixed value of the encoding parameter ($\gamma = 0.004$) that produces reasonably low MPE's for the two sets of test imagery used. A HC filter and its pseudorandom encoding are illustrated in Fig. 2. The left side shows the gray-scale magnitudes of the filter. The right side of the figure has been binarized to indicate the pixels (in white) that are random encoded.

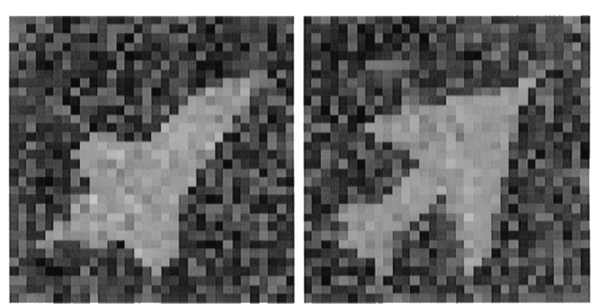

Fig. 1. One of the 36 views of the test imagery: target (Space Shuttle, left) and one of three clutter objects (Phantom, right). The noise was included in the test images only for case a of Table 1.

Table 1. Filter Bank MPE (%) for Bank Size and Type of Test Imagery

Bank Size	FCF	POF	PRF[γ]	PRF[0.004]
Case a, noisy test images				
1	22	28	27 [0.003]	28
2	7	9	6 [0.004]	6
3	6	4	2 [0.008]	3
4	1	4	1 [0.008]	1
5	4	2	0 [0.003]	1
Case b, noise-free, angle-offset test images				
1	31	40	37 [0.07]	41
2	17	28	20 [0.07]	23
3	20	25	17 [0.07]	22
4	11	24	17 [0.06]	17
5	9	23	15 [0.07]	18

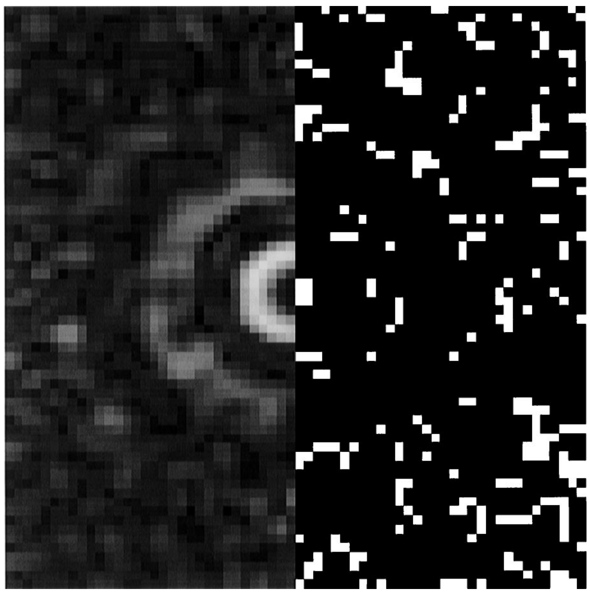

Fig. 2. Gray-level magnitudes of one HC filter. The right-hand side of filter has been binarized, indicating in white the pixels that are pseudorandom encoded for $\gamma = 0.004$.

The filter corresponds to filter number 1 in Table 1 and has a diffraction efficiency η of 4.6%. As the filter is quite nearly symmetric, the pattern of random encoding at the right of the figure will appear quite similar. For this encoding of the filter, $\gamma = 0.004$, $N_r = 480$ of the $N = 4096$ total pixels, and the maximum amplitude randomly encoded is 0.057 of the maximum (i.e., unity) filter amplitude.

To demonstrate the performance that results from different encodings we tested the filter banks against degraded imagery. In Table 1, case a, additive white Gaussian noise is added to each test image. Typical images are shown in Fig. 1. The total signal-to-noise ratio is 4:1, or 6 dB. The MPE for the 36 views is calculated 10 times, each time with a change of only the random seed for the scene noise generator. The average MPE of this test is reported in the table. In all cases the PRF has equal or lower MPE than the POF. This is true even for the case of the fixed encoding parameter $\gamma = 0.004$. However, in some cases the MPE for the PRF, and even for the POF, is lower than that for the FCF. These crossovers are not inconceivable when one recognizes that α_1 and α_2 have been selected to give best overall performance for various scene distortions, noise, and clutter and are not necessarily the optimal choices, for any one environment. Therefore phase-only encoding could, by chance, have lower MPE.

In case b the test images are distorted views of the original training and clutter objects. The views are taken with a fixed angular offset in azimuth from the original 36 views. The MPE is calculated for each offset, 1° to 9°, in 1° increments, and the average of the MPE's is presented in Table 1. With larger MPE's the filter bank learning curves appear more stable and the differences are more easily seen. For more than one filter, both PRF columns have lower MPE than the POF. The PRF with optimized MPE for two and three filters has MPE that is comparable with that for the FCF.

An additional trend noted in Table 1 is that the encoding parameter γ appears to increase as the discrimination task becomes more challenging. Case b, which has the most errors for a given number of filters, typically has the largest values of γ. This means that more of the pixels are pseudorandom encoded for this case. Further study is needed to determine whether the trend is reliable and, if it is, the reason for it.

We have shown that pseudorandom encoding the lowest amplitude pixels of composite, fully complex filters can noticeably improve recognition performance over that which is possible with the traditional phase-only encoding. Although pseudorandom encoding does not produce the optimal mapping for correlation, it does provide some level of improvement in recognition and is easy to use with real-time hardware. For the study we did searches to find the best threshold for random encoding. However, a fixed encoding threshold can often produce improved recognition.

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