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Periodic envelope of Coulomb-blockade oscillations in the quantum Hall regime

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The irregular envelope of Coulomb-blockade oscillations in the conductance of a quantum dot as a function of gate voltage is observed to change into a *periodic* modulation in quantizing magnetic fields. The period corresponds to depopulation of each of the Landau levels in the dot by a single electron, with tunneling occurring predominantly through a single edge state of the outermost Landau level. Magnetoconductance data are consistent with this interpretation, which implies that the energy spectrum has an unexpected regularity.

The conductance of a quantum dot that is weakly coupled to leads by tunnel barriers allows a study of the interplay of charge quantization [leading to the Coulomb blockade (CB) of tunneling], size quantization (leading to a discrete energy spectrum), and magnetic quantization [leading to the formation of Landau levels (LL's)].[1-6] At temperatures T such that $k_BT \ll e^2/2C$, with C the capacitance of the dot, conductance oscillations are observed as a function of the voltage applied to a capacitively coupled gate electrode. These oscillations, also known as CB oscillations,[7] correspond to depopulation of the dot by a single electron per period. Whereas the periodicity of the CB oscillations is well understood, it is less clear what causes the variations in amplitude from peak to peak that are observed. We will refer to these variations as the *envelope* of the CB oscillations. Recently, Jalabert, Stone, and Alhassid have explained the occurrence of irregular envelopes in zero or weak magnetic field in terms of chaotic fluctuations of the tunnel rates.[8]

Here, we study the envelope of the CB oscillations in the quantum-Hall effect regime. We find that it exhibits a *periodic* modulation, each period consisting of the same number of CB oscillations. We argue that this surprising effect is due to electrostatic depopulation of each of the LL's in the dot by a *single electron* per period, with tunneling occurring predominantly through a single edge state of the outermost (lowest) LL. A model calculation and additional magnetoconductance data are used to support this interpretation.

Our device is shown schematically in Fig. 1. We used a GaAs-$Al_xGa_{1-x}As$ heterostructure containing a two-dimensional electron gas (2DEG) with electron-density $n_s = 3.7 \times 10^{11}$ cm^{-2} and mobility $\mu \approx 10^6$ cm^2/V s. Part of the 2DEG is confined electrostatically to a quantum dot by patterned Ti-Au gates on top of the heterostructure. The tunnel barriers defined by gates A, B, and D were adjusted to a zero-field conductance of approximately $\frac{1}{2}e^2/h$. The sample was mounted in the mixing chamber of a dilution refrigerator, and two-terminal conductance measurements were made using a standard lock-in technique with an excitation voltage less than 9 μV, both as a function of gate voltage (applied to gate C) and magnetic field B.

In Fig. 2, traces of the CB oscillations as a function of gate voltage are shown at $T = 45$ mK and four values of the magnetic field. A large number of conductance peaks is observed in the traces, each peak corresponding to the electrostatic depopulation of the dot by a single electron. In contrast to the period of the CB oscillations, which is rather insensitive to a magnetic field, the amplitude exhibits a pronounced B dependence. Whereas in the absence of a magnetic field the envelope varies irregularly, in the quantum Hall effect regime it oscillates *periodically*, in the sense that at fixed B *each period contains the same number of conductance peaks*, and thus corre-

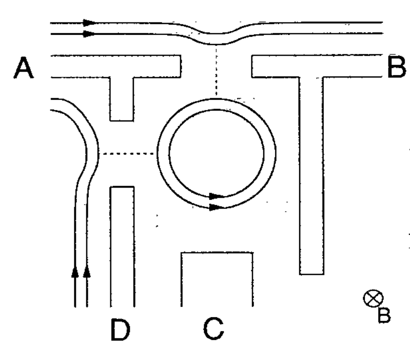

FIG. 1. Schematic top view of the 0.7×0.8 μm^2 quantum dot. Gates A, B, and D (hatched) define individually adjustable tunnel barriers, and gate C controls the electrostatic potential of the dot; the gaps between gates B and C, and between gates C and D, are pinched off in the experiment. The drawn lines represent edge channels formed by a quantizing magnetic field B perpendicular to the plane of the 2DEG, and the dashed lines indicate tunneling paths between the outermost LL in the dot and the leads.

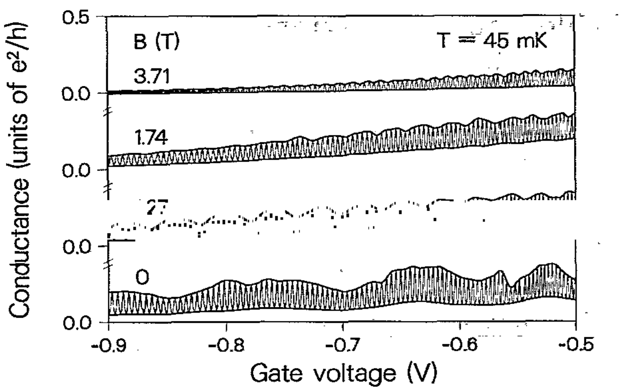

FIG. 2. Conductance as a function of the voltage applied to gate C at $T = 45$ mK and four values of B. The envelope is indicated by the thick lines.

sponds to the same number of electrons removed from the dot. This is true even though the period increases slowly as the gate voltage is made more negative, presumably because the size of the dot is decreased (thereby decreasing the mutual capacitance of dot and gate C). On increasing the magnetic field, the period decreases from nine conductance peaks per period at $B = 1.27$ T down to three peaks at 3.71 T. As the temperature is increased to 0.5 K, the envelope oscillations disappear (see lower right inset in Fig. 3), whereas the CB oscillations remain clearly visible (upper left inset). The charging energy $e^2/2C$ of the dot may be estimated from the self-capacitance $C = 4\varepsilon_0\varepsilon_r d$ of a flat circular disk of diameter d. In our experiment $d \approx 0.5$ μm (based on the assumption that the lateral depletion length equals half the lithographical width of the tunnel-barrier openings), so that $e^2/2C \approx 0.4$ meV (using ε_r=13), consistent with the disappearance of the CB oscillations at $T \approx 4$ K.

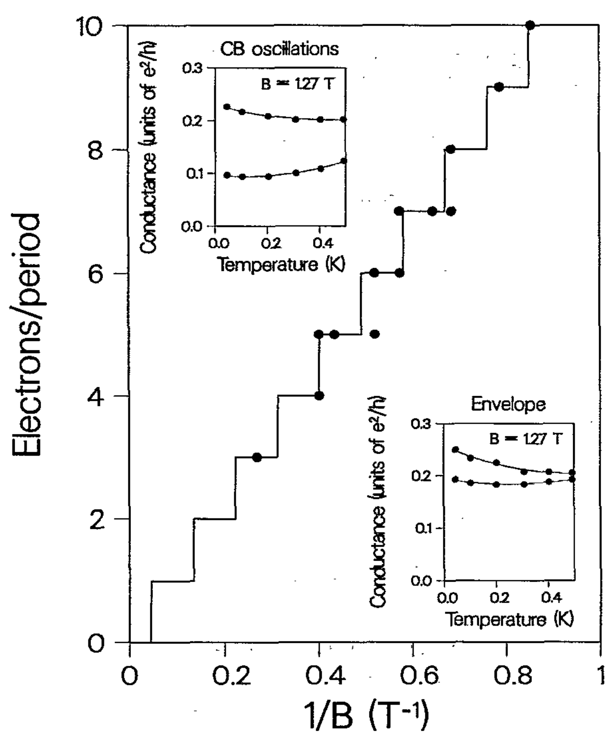

FIG. 3. Number of conductance peaks per period of the envelope vs $1/B$ (filled circles). The drawn curve indicates the number of LL's in the dot. Insets: temperature dependence of the maxima and minima of the CB oscillations (upper left) and envelope oscillations (lower right), averaged over a large number of periods. The drawn lines are guides for the eye.

The *period* of the envelope oscillations (in terms of the number of electrons removed from the dot) corresponds to the number N_L of LL's occupied in the dot. This is evident from the plot in Fig. 3, showing the number of conductance peaks (and thus electrons) per period of the envelope oscillations versus $1/B$. The drawn staircase curve results from a fit of the data to the filling factor $hn_{\rm dot}/eB$ rounded to the nearest integer, which is equal to the number of spin-resolved LL's in the dot (treated as a 2DEG for simplicity). The value thus obtained for the electron density, $n_{\rm dot} = 2.7 \times 10^{11}$ cm^{-2}, is about 25% smaller than the density of the ungated regions, which is quite reasonable.

We now argue that our experiment reveals a surprising aspect of the energy spectrum of the quantum dot. Our observations can be explained if (a) the LL's are depopulated cyclically, by a single electron per LL per period of the envelope oscillations, and (b) only the outermost (lowest) LL in the dot couples to the leads. The first assumption is not a trivial one, and is in contrast the sequential depopulation from highest to lowest LL in an unconfined 2DEG if the Fermi energy is reduced. Since the number of electrons removed from the dot per period is equal to N_L, we require that in between each two states of the lowest LL a single state of each of the remaining occupied LL's is present. Such a regularity of the spectrum is unexpected, but we know of no other way to account for the periodically modulated envelope with the particular period observed.

Cyclic depopulation of the LL's by itself does not explain why the envelope of the CB oscillations would be modulated. This requires an additional argument. The simplest explanation for a small modulation would be small differences in tunnel rates for states of different LL's. However, the tunnel rates through a split-gate barrier (or quantum-point contact) are known to decrease exponentially with increasing LL index.[9] This motivates our second assumption given above. Consecutive conductance peaks then result from tunneling through the nearest state (in energy) of the lowest LL, with an amplitude determined by the occupation probability of that state (which is a function of temperature and Fermi energy in the dot). The states of the remaining LL's accommodate additional electrons induced electrostatically in the dot, but do not provide a tunneling path. Their presence affects the position of the Fermi level in the dot, and thus (indirectly) the tunneling probability. This is what explains the existence of an envelope.

In order to verify the validity of our arguments, we have made some calculations using the theory of Ref. 3 and the spectrum of noninteracting electrons in a dot with a parabolic confining potential of strength ω_0, for which the energy levels within each single LL are equally spaced (independent of the LL index n):[10]

$$E_{n,m} = \tfrac{1}{2}(n-m)\hbar\omega_c + \tfrac{1}{2}\hbar(\omega_c^2 + 4\omega_0^2)^{1/2}(n+m-1) \, , \tag{1}$$

with $\omega_c = eB/m^*$ the cyclotron frequency of electrons with effective mass m^*, and m the index of consecutive states within a LL. [Spin is ignored in (1) for simplicity.] As shown in the top panel of Fig. 4, at fixed B this spectrum is periodic in E, with period $\Delta E = E_{n,m} - E_{n,m-1} = -\frac{1}{2}\hbar\omega_c + \frac{1}{2}\hbar(\omega_c^2 + 4\omega_0^2)^{1/2}$ equal to the spacing of states within each single LL (independent of n and m). It therefore satisfies the above regularity criterium. In the bottom panel of Fig. 4 results of the calculations are given for a value of B indicated in the top panel by the dashed line, corresponding to a spectrum of nearly equidistant levels with $N_L = 4$. Clearly, the amplitude of the CB oscillations exhibits an envelope with a period that is determined by the number of LL's in the dot (left part). The modulation disappears as k_BT approaches ΔE (right part). At very low temperatures, such that $k_BT \ll \delta E \approx \Delta E/N_L$, only a single peak remains per period of the envelope, associated with tunneling through a state of the lowest LL. In this regime, which was not accessible in our experiment, the other peaks are suppressed by orders of magnitude (not shown). We point out that the amplitude of the calculated envelope is not very sensitive to the distribution of the energy levels in between two consecutive states of the lowest LL. In our calculation we have found an increase by only about 10% if B is chosen such that the energy levels are fourfold degenerate. The position of the envelope oscillations, however, shifts over a complete period if the magnetic field is increased by ΔB, corresponding to the nearly periodic character of the spectrum as a function of B at fixed E (see top panel of Fig. 4).

Experimentally, we also observe a shift of the envelope oscillations as the magnetic field is changed slightly. In Fig. 5 it is shown that the envelope is shifted over a full period if the magnetic field is increased by $\Delta B \approx 20$ mT. As shown in the inset, this value is equal to the period of the magnetoconductance oscillations observed at fixed gate voltage (adjusted to the maximum of a zero-field conductance peak). The amplitude and activation energy of the magnetoconductance oscillations are approximately equal to those of the envelope oscillations, further illustrating their common origin. Note that the fact that ΔE, and not $e^2/2C$, governs the activation energy of the magnetoconductance oscillations implies that the conductance remains at a maximum of a CB conductance peak as the magnetic field is changed. We have not attempted a direct comparison of experimental and calculated conductance as a function of magnetic field, because of the sensitivity to the B dependence of the Fermi level in the leads, which is not easy to model reliably.

The results of the calculations using the single-electron energy spectrum (1) are in qualitative agreement with our experimental observations. We have traced the ori-

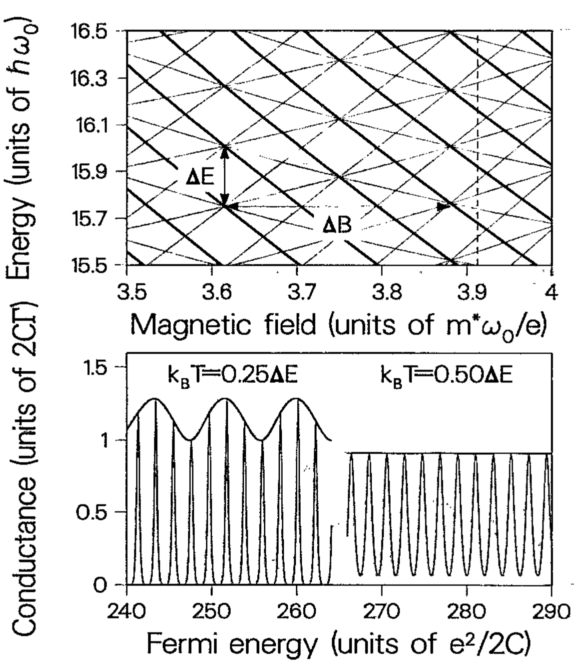

FIG. 4. Top panel: spectrum of noninteracting electrons in a parabolic confining potential, which is periodic in E with period ΔE, and quasiperiodic in B with period ΔB. The states of the lowest LL are indicated by the thick lines. Bottom panel: model calculations using the theory of Ref. 3 for a value of the magnetic field indicated in the top panel by the dashed line (using $\hbar\omega_0 = 0.6$ meV, $e^2/2C = 0.4$ meV, and Γ and $10^{-2}\Gamma$ for the tunnel rates of the states of the lowest LL and remaining LL's, respectively).

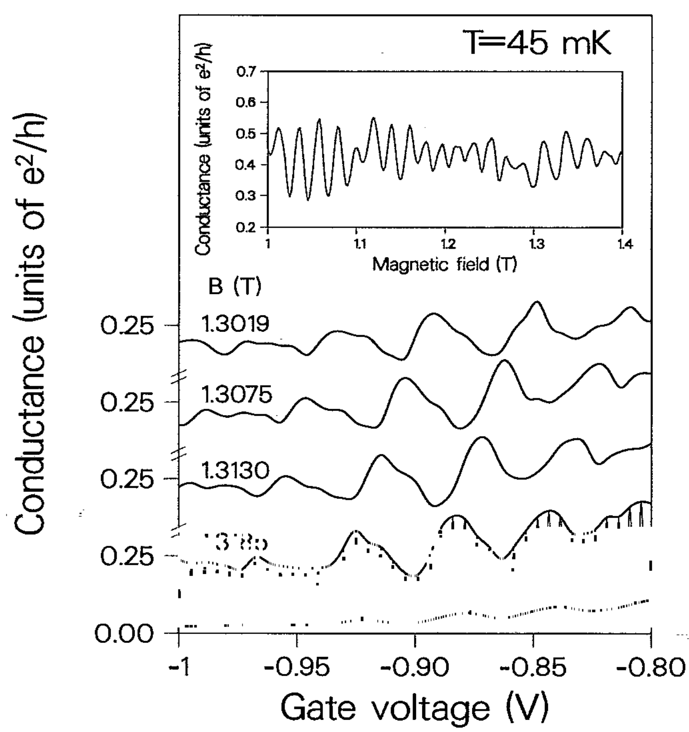

FIG. 5. CB oscillations (shown for the bottom trace only) and envelope at $T = 45$ mK and four values of B. Inset: magnetoconductance oscillations at a gate voltage of -0.725 V.

gin of the agreement to the regularity of the energy spectrum, which leads to a *cyclic depopulation of the LL's by one electron at a time.* McEuen *et al.*[5] have recently shown that the self-consistently calculated energy spectrum differs substantially from that of noninteracting particles. Our experiment suggests that the regularity leading to cyclic depopulation of LL's is a *generic* feature of a strongly interacting electron gas in a quantum dot. It would be interesting to investigate if, or to what extent, this is borne out by the self-consistent model presented in Ref. 5.

In conclusion, we have investigated the envelope of the CB oscillations as a function of gate voltage in the conductance of a quantum dot. The envelope is shown to exhibit periodic oscillations in the quantum Hall effect regime, with a period corresponding to the electrostatic depopulation of each of the LL's in the dot by a single electron. We have argued that this effect can be understood by assuming that only edge states of the outermost (lowest) LL in the dot couple to the leads, and that the LL's in the dot are depopulated cyclically. The latter assumption implies that the self-consistent spectrum of the quantum dot is regular, in the sense that in between two states of the lowest LL a single state of each of the remaining LL's exists. Such a regularity of the spectrum is unexpected for a generic confining potential.

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