

Magnetic-field dependence of the quantum-Hall-liquid-insulator transition

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We present measurements of the magnetic field dependence of the insulator transition in a gated $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterostructure. The resulting phase diagram shows reentrant behavior as the magnetic length decreases below the zero-field electron mean free path. By fitting our data to a predicted phase diagram, we obtain a value for the localization length exponent governing the quantum-Hall-liquid-insulator transition. As has been recently suggested, this value is in agreement with that determined for the critical exponent between quantum Hall plateaus.

The metal-insulator transition for noninteracting electrons in a disordered system has been studied extensively.¹ In 1984 Shapiro² proposed a phase diagram incorporating the two main influences of magnetic field on the metal-insulator transition. At low magnetic field, where the zero-field electron mean free path l_e is much less than the magnetic length $l_B = \sqrt{\hbar/eB}$, magnetic field shifts the mobility edge E_c towards lower energies due to phase breaking of weak-localization pathways. The magnetic field B_c at the mobility edge in the low field regime is given by

$$B_c \sim |E_F - E_c|^{2\nu}, \quad (1)$$

where ν is the critical exponent of the localization length ξ near E_c .³ A strong magnetic field ($l_e \gg l_B$), however, increases the localization by shrinking the radius of the bound states. In the high field regime,

$$B_c \ln B_c \sim |E_F - E_c|^{3/2}. \quad (2)$$

Shapiro suggested that due to these two effects it is possible for an insulating sample at $B=0$ to transform first into a metal and then back into an insulator with monotonically increasing magnetic field. Evidence for this insulator-metal-insulator transition has been observed experimentally in disordered three-dimensional (3D) semiconductors.⁴

Recently, Kivelson, Lee, and Zhang⁵ (KLZ) have proposed a phase diagram for the transition to the insulating state in a quantum Hall conductor. It is well known that the localization length in a quantum Hall conductor diverges near the Landau level energies E_n according to $\xi \sim |E_F - E_n|^{-\nu}$.⁶ For increasing levels of disorder the delocalized energy levels rise and eventually, the quantum Hall liquid transforms into what is called a quantum Hall insulator. The quantum Hall insulator is characterized by a longitudinal resistance that tends to infinity while the Hall resistance remains near its normal value as temperature tends to zero.⁷ KLZ predict that for a certain level of disorder, a sample which behaves as a quantum Hall insulator at low fields can transform into a quantum Hall liquid as the lowest Landau level energy moves through E_F with increasing magnetic field. As the field continues to increase through the Hall plateau, the delocalized states rise above the Fermi energy and a transition back to a quantum Hall insulator takes place. This quantum-Hall-insulator-liquid-insulator transition has

been used to explain the reentrant behavior of the longitudinal resistance that has recently been observed in highly disordered conductors.⁸⁻¹¹

KLZ stress that their theory is only valid in the high field regime where $l_e \gg l_B$; the crossover from the high to the low field regime has yet to be explained. Although the phase diagram described by Shapiro takes both magnetic-field regimes into account, it is unclear whether this 3D theory may be used to describe a 2D quantum Hall conductor. To this end we have performed experiments that allow us to map out the transition to the insulator state in a quantum Hall conductor as magnetic field increases from the low to the high field regime. We determine the mobility edge E_c at a particular magnetic field by measuring the two-terminal conductance versus electron density at a number of different temperatures. In this way, we are able to produce a phase diagram for the quantum-Hall-liquid-insulator transition for magnetic fields ranging from 0.1 to 4.0 T. This diagram reveals a metal-insulator-metal transition of the kind described by Shapiro, and to our knowledge is the first such diagram that has been experimentally produced in any system. By fitting our diagram to Eqs. (1) and (2) we obtain a value for the critical exponent of $\nu=2.4$. This is in good agreement with recent theoretical¹² and experimental¹³ work implying (as is suggested in Ref. 5), that the quantum-Hall-liquid-insulator transition has the same character as the transition between quantum Hall plateaus.

We performed two-terminal conductance measurements of a gated Hall bar made from a standard $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterostructure with magnetic field directed perpendicular to the interface. The $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layer is 60 nm thick with two central $5 \times 10^{12} \text{ cm}^{-2}$ delta-doped layers of Si. The resulting two-dimensional electron gas (2DEG) has a density $1.67 \times 10^{11} \text{ cm}^{-2}$ and a mobility of $8.6 \times 10^5 \text{ cm}^2/\text{V s}$. The gate is macroscopic in size with dimensions of $0.26 \times 0.1 \text{ mm}^2$. A conventional ac lock-in technique was used with an excitation voltage of less than 5 μV . Additional probes allow two-terminal measurements of the 2DEG on either side of the gate, however we are unable to make four-terminal measurements of the gated region of this device structure. Three devices were measured from the same wafer, each showing similar characteristics. Measurements taken from one of the three devices is presented in this paper.

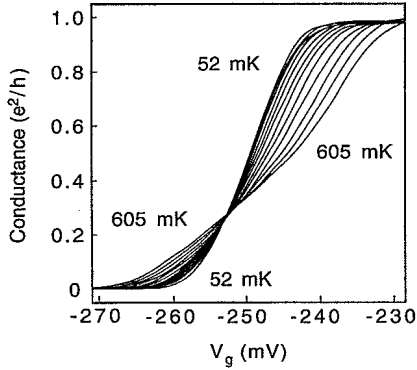


FIG. 1. Two-terminal conductance as a function of gate voltage V_g for $T=52, 80, 109, 124, 142, 168, 198, 247, 303, 370, 466,$ and 605 mK at $B=1.65$ T.

Figure 1 shows the two-terminal conductance as a function of gate voltage V_g at 12 temperatures between 50 and 600 mK. The measurements are at a magnetic field of $B=1.65$ T and correspond to a filling factor approximately equal to 4 for $V_g = 0$. The conductance of the gated region decreases from a plateau of e^2/h down to shut-off as the electron density underneath the gate decreases with increasing bias. A critical gate voltage V_{gt} is clearly seen in the temperature dependence of the conductance at -253 mV: for $|V_g| > |V_{gt}|$ the conductance increases with increasing temperature, while for $|V_g| < |V_{gt}|$ the conductance decreases with increasing temperature. We point out that although the resistances of the 2DEG leads are in series with the gated region, their resistance changes by less than 400Ω as a function of temperature between 50 and 600 mK. This is less than 1% of the resistance at the transition point—an amount within experimental error—and thus does not affect our ability to determine V_{gt} .

The temperature dependence of the conductance is shown explicitly in Fig. 2. Here, the two-terminal conductance is plotted on a logarithmic scale as a function of $T^{-1/3}$ for three values of V_g . For $|V_g| > |V_{gt}|$ the data follows Mott's well known law $G \sim \exp[-(T_0/T)^{1/3}]$, with a fit giving $T_0 \approx 1$ K. This demonstrates that the 2DEG behaves as an Anderson

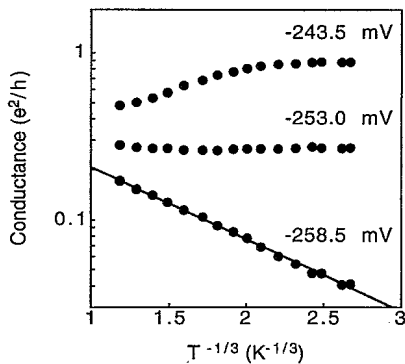


FIG. 2. Logarithmic plot of the two-terminal conductance as a function of the cube root of the inverse temperature. The three values of the gate voltage correspond to points above, at, and below the insulator transition.

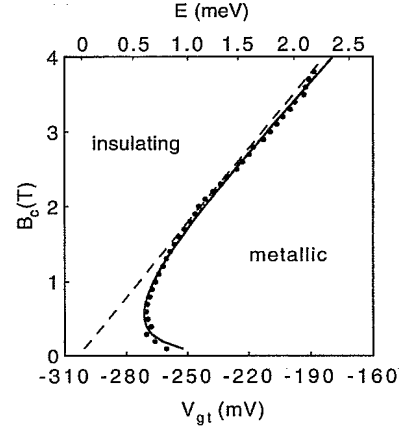


FIG. 3. Experimentally determined relationship between the transition gate voltage (or transition energy) and the magnetic field. To the left (right) of the points the sample behaves as an insulator (metal). The solid line is a best least-squares fit to the theoretical phase diagram proposed by Shapiro (see text), while the dashed line corresponds to a filling factor of $2/3$ underneath the gate.

insulator in this regime. For $|V_g| = |V_{gt}|$ the conductance is almost temperature independent while for $|V_g| < |V_{gt}|$ the conductance increases with decreasing temperature and approaches a plateau at e^2/h , behavior characteristic of a quantum Hall liquid. We observe a transition of this kind at a wide range of magnetic field strengths. In Fig. 3, V_{gt} determined from a series of measurements similar to those plotted in Fig. 1 is shown for $B=0.1-4.0$ T. $|V_{gt}|$ first increases, then decreases with magnetic field; the turning point occurs near $B=0.5$ T. At higher magnetic fields, $|V_{gt}|$ decreases almost linearly with B .

The data suggest that the transition point in gate voltage V_{gt} corresponds to a quantum-Hall-liquid-insulator transition. As the gate voltage becomes more negative, the Fermi energy of the underlying electron gas decreases through the lowest defined Landau level. In addition, the disorder potential effectively increases with decreasing electron density causing the delocalized states to float up in energy.¹⁴ The transition voltage corresponds to the energy where E_F crosses the delocalized states. For $|V_g| < |V_{gt}|$, transport occurs through the disorder broadened extended states associated with the lowest energy resolved Landau level. For $|V_g| > |V_{gt}|$ transport occurs via hopping conduction among the localized states.

We transform the gate voltage axis of Fig. 3 into an energy axis by using measurements of the quantum Hall plateaus as a function of magnetic field (not shown) to determine the electron density and thus the Fermi energy as a function of gate voltage.¹⁵ Figure 3 can then be viewed as a phase diagram of the magnetic-field dependence of the mobility edge in a quantum Hall conductor. For energies to the right of the experimental points, the system behaves as a metal (or a quantum Hall liquid) while for energies to the left the system behaves as an insulator. Our experimentally determined phase diagram is strikingly similar to the diagram proposed by Shapiro for the general case of noninteracting electrons in a disordered system.² This is perhaps not surprising: as the density decreases, the elastic scattering length

decreases until weak localization effects near zero field become important. Application of a weak magnetic field then decreases the phase-breaking length, and reduces the weak localization. As described above, this is the same low field mechanism considered in Shapiro's theoretical phase diagram.

As the magnetic field increases above the low field regime, E_c must eventually increase. The turning point in Fig. 3 should thus approximately correspond to the point where the magnetic length l_B decreases below the zero-field elastic scattering length l_e . We can estimate l_e from the conductivity and the electron density underneath the gate near the insulator transition. For $B=0$ and $V_g = -250$ mV this gives $l_e \approx 3.8 \times 10^{-8}$ m which is equal to l_B at $B=0.43$ T. This is reasonably close to the turning point we observe at $B=0.5$ T.

By taking a linear combination of Eqs. (1) and (2), we obtain the following expression for the Fermi energy at the insulator transition:

$$E_F \sim [E_c + (B_c \ln B_c)^{2/3}] + (E_c - B_c^{1/2\nu}). \quad (3)$$

In order to fit this expression to the data in Fig. 3 we assume that E_c is a constant and that $E_F \sim V_{gt} - V_{gt}(0)$ where $V_{gt}(0)$ is the transition gate voltage at $B=0$. The solid line in Fig. 3 shows a best least squares fit of our data to the resulting equation:

$$V_{gt} - V_{gt}(0) = C_1 (B_c \ln B_c)^{2/3} + C_2 B_c^{1/2\nu}, \quad (4)$$

where C_1 , C_2 are fitting parameters. The fit matches the data very well for $C_1=7.48$, $C_2=115$, and $V_{gt}(0)=-142$ mV and gives a value for the localization length exponent of $\nu=2.4$.

Recently, it has been suggested that the transition between quantum Hall plateaus should have the same character as the quantum-Hall-insulator transition.^{16,5} This is because in both cases the transition is defined by the point at which the delocalized states float up above E_F ; in the quantum-Hall-liquid-insulator transition the delocalized states are simply associated with the lowest, rather than one of the higher order Landau levels. Recent theoretical¹² and experimental¹³ predictions for the critical exponent governing the transition between quantum Hall plateaus give values of 2.34 ± 0.04 and 2.3 ± 0.1 , respectively. Our value for the critical exponent determined from data in the low field regime is in agreement with these values. This is possibly fortuitous: the value for $V_{gt}(0)$ seems rather low and additional measurements are needed of samples with various amounts of disorder to test the generality of this result.¹⁷ Nevertheless, the fact that the critical exponent determined here for the insulator transition matches that for the divergence of the localization length between quantum Hall plateaus lends support to the claim that the two transitions are in fact quite similar, even in the low field regime where $l_e < l_B$.

Between 4 and 6.4 T, the filling factor at $V_g=0$ decreases from 2 to 1. In this regime, the temperature dependence of the leads is no longer negligible, and we are unable to determine $V_{gt}(B)$ accurately. Between 6.4 and 7.4 T, however, the lead conductance is quantized at the e^2/h plateau, and is independent of temperature between 50 and 600 mK. Here, two-terminal measurements are again a valid means to determine the quantum-Hall-liquid-insulator transition point.

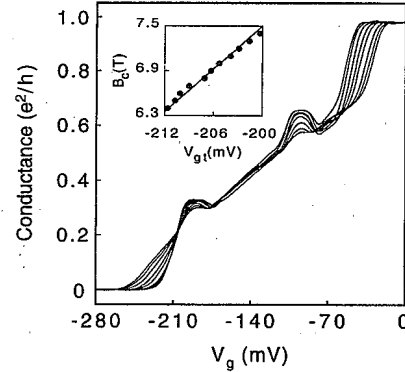


FIG. 4. Two-terminal conductance as a function of gate voltage V_g for $T \approx 50, 100, 150, 200, 250, 350, 450,$ and 550 mK at $B=6.9$ T. Inset: Relationship between the magnetic field and the transition gate voltage in the high field regime. The solid line corresponds to a filling factor of $1/5$ underneath the gate.

Figure 4 shows the two-terminal conductance as a function of V_g at $B=6.9$ T (corresponding to a filling factor of 1 for $V_g=0$) for eight temperatures between 50 and 600 mK. Well defined plateaus are observed at e^2/h , $2/3(e^2/h)$, and $1/3(e^2/h)$. A transition point is also seen in the temperature dependence of the conductance at $V_{gt} = -206$ mV where the conductance is approximately $0.2(e^2/h)$. In the inset to Fig. 4, V_{gt} (determined from a series of measurements similar to those shown in the main figure) is plotted as a function of magnetic field. $|V_{gt}|$ decreases linearly with B , indicating that the filling factor underneath the gate is constant at the insulator transition. We note that this is also true of the transition observed in the high field region of Fig. 3: as the magnetic field approaches 4 T, $|V_{gt}|$ decreases nearly linearly as a function of B .

We can determine the specific filling factors for the transitions in each of the two linear regimes by extrapolating from the gate voltage positions of the quantum Hall plateaus. The dashed line plotted in Fig. 3 is an extrapolation of the magnetic-field dependence of the gate voltage position of the $2/3(e^2/h)$ plateaus observed for magnetic fields greater than 6.4 T. This line fits the linear magnetic field dependence from 1.5 to 4.0 T reasonably well, indicating that the filling factor underneath the gate is approximately $2/3$ at the transition voltage. By decreasing the slope of this line, we obtain a fit to the high field regime in Fig. 4: the line drawn in the inset to Fig. 4 has a slope corresponding to a filling factor of $1/5$, and is a fairly good fit to the magnetic-field dependence of the transition voltage.

These results show that the insulator transition occurs at filling factors which at higher fields would correspond to well-defined fractional quantum Hall states. As magnetic field increases to 4 T (Fig. 3) the filling factor at the insulator transition approaches $2/3$. As the field continues to increase, the $2/3(e^2/h)$ plateau becomes resolved, and extended states associated with the $2/3$ fractional Hall state come into existence. In this regime, V_{gt} must cross over to a lower filling factor until finally, when the $2/3(e^2/h)$ and $1/3(e^2/h)$ plateaus are completely resolved (Fig. 4), the insulator transition is observed at a filling factor of $1/5$. It seems likely that at higher magnetic fields where the $1/5$ quantum Hall state is

resolved, the insulator transition will again cross over to a lower filling factor. Unfortunately, we are unable to reach this high field regime in our experiment.

It is intriguing that the insulator transition occurs at specific filling factors before these states are resolved in the Hall conductance. In the KLZ model,⁵ a continuous quantum-Hall-liquid-insulator transition occurs only through a specified series of filling factors starting with 1, 1/3, and 1/5. In comparison, we observe transitions through 2/3 and 1/5, while in previously reported experiments on highly disordered samples the insulator transition is observed at a filling factor of 2.^{8,11} As stated in Ref. 8 it is possible that at low fields and high levels of disorder spin is not resolved, and transitions would be expected at 2 times the values predicted by KLZ.⁵ This would suggest that at the 2/3 transition (shown in Fig. 1) the effective disorder has increased to the extent that spin—which is resolved at zero gate voltage—is no longer resolved. However, at the 1/5 transition (Fig. 4) the magnetic field is high enough for spin to be resolved.

In conclusion, we have experimentally determined the magnetic-field dependence of the quantum-Hall-liquid-insulator transition. At low magnetic fields, reentrant behavior is observed that fits within Shapiro's theoretical description of noninteracting electrons in a disordered system. This gives a critical exponent for the localization length very close to the critical exponent governing the transition between quantum Hall plateaus, implying that the two transitions are in fact quite similar. We also show that the insulator transition in our relatively high mobility sample occurs near fractional filling factors of 2/3 or 1/5. Further experiments are needed to generalize these results to higher magnetic fields and to samples with different amounts of disorder.

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³Traditionally, ν has been used to refer to both the critical exponent and the filling factor. To avoid confusion, we use ν to refer to the critical exponent, and simply write out "filling factor."

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¹⁷In fitting the data we have assumed that the electron energy is proportional to the gate voltage. This implies that the density of states at E_F is constant at the transition point which is not strictly correct. A more exact treatment is necessary to determine the critical exponent precisely.