New Resistivity for High-Mobility Quantum Hall Conductors

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We present measurements showing dramatic nonlocal behavior in the four-terminal resistances of a high-mobility quantum Hall conductor. These measurements illustrate that the standard definition of the resistivity tensor is inappropriate, but they are in excellent agreement with a new model of the conductor that treats the edge and bulk conducting pathways independently. This model uses a single intensive parameter, analogous to a local resistivity for the bulk channel only, to characterize the system.

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Since the discovery of the quantized Hall effect in two-dimensional electronic systems,[1] much attention has been paid to the transition regions between plateaus in the Hall resistance and the concurrent Shubnikov–de Haas (SdH) oscillations in the longitudinal resistance. In low-mobility samples ($\mu \lesssim 100000$ cm^2/Vs in GaAs, $\mu \lesssim 30000$ cm^2/Vs in Si), these measurement are relatively well understood and are interpreted in terms of a resistivity tensor derived from the Hall and longitudinal resistances through the following relations: $R_H = \rho_{xy}$ and $R_L = \rho_{xx}(L/W)$, where L and W are the length and width of the device. However, measurements on high-mobility samples have produced many surprising results that are not well understood. Most striking is the fact that the longitudinal resistance does not scale linearly with L/W.[2-4] Kane, Tsui, and Weimann[4] suggested that edge currents play an important role, and recent experiments[5-7] have clarified that suppression of scattering between the edge and bulk current-carrying channels is necessary to observe this behavior. Many authors[5,8-10] have since argued that in these systems the standard definition of the resistivity tensor is not applicable, and it is not known how to describe the conductor by an intensive, i.e., geometry-independent, quantity.

In this Letter, we show that while the measured resistances of high-mobility samples show unusual behavior, a single intensive quantity can be used to describe the data in a consistent manner. We begin with experiments demonstrating that the four-terminal resistances are nonlocal; i.e., they depend upon properties of the entire conductor. This produces nonintuitive behavior: Measurements yield different results if the probes are different, and voltages appear between probes far removed from the classical current path. We argue that these nonlocal effects arise when there is suppression of interchannel scattering between edge and bulk current-carrying channels. We than apply a model[11] that describes the conductor by a new intensive parameter analogous to the longitudinal resistivity, but related to only the topmost Landau level. We find excellent quantitative agreement with this model. Finally, we derive the new resistivity from the experimental data and discuss how it relates to the properties of the bulk Landau level.

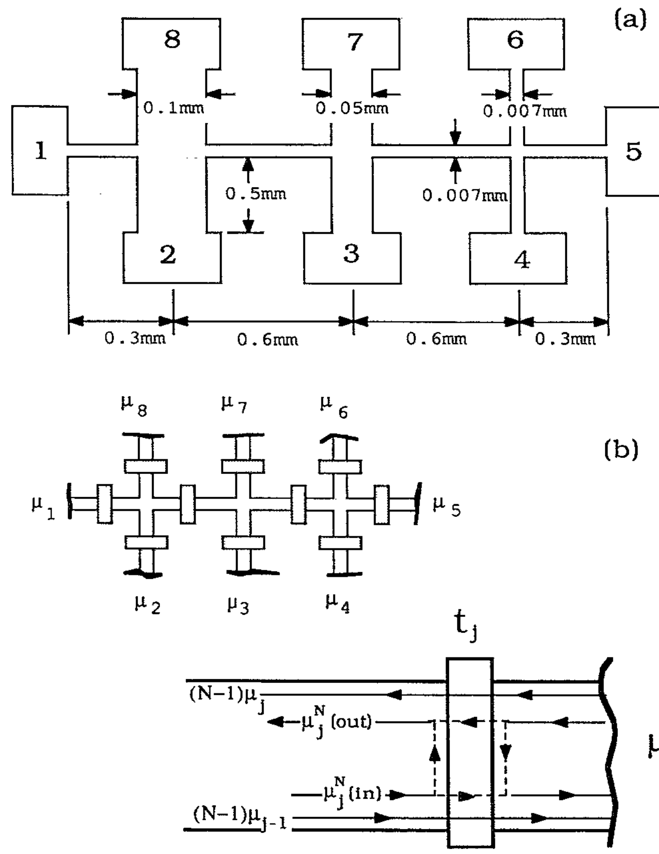

FIG. 1. (a) Schematic of the device used in these experiments. (b) A model of the conductor where each segment is represented by a barrier that only backscatters the Nth channel.

The device used in the experiments is schematically shown in Fig. 1(a). It was defined by wet etching of a standard high-mobility GaAs/AlGaAs heterostructure with $n_s = 3.5\times10^{11}/\mathrm{cm}^2$ and $\mu = 1\,200\,000\ \mathrm{cm^2/V\,s}$. The measurements are made by standard low-frequency lock-in techniques with a measurement current of 20 nA. The temperature is $T = 0.45$ K unless otherwise specified, and the direction of the magnetic field is such that electrons in the edge channels circulate counterclockwise. We first consider longitudinal resistance measurements where current is passed between probes 1 and 5 and voltage is measured between the lower probes 3 and 4 ($R_{15,34}$), or between the upper probes 7 and 6 ($R_{15,76}$). In both cases, the segment of the conductor between the probes is the same, and the measurements agree at low magnetic fields [Fig. 2(a)]. However, at higher fields the two measurements deviate from each other substantially. An even more striking result is shown in Fig. 2(b). In the measurement $R_{37,46}$ ($R_{37,28}$), current is passed between probes 3 and 7, and voltage is measured between probes 4 and 6 (2 and 8) that are connected to the apparent current pathway by a single narrow segment 0.6 mm long by 0.007 mm wide. At low fields, the voltage between these probes is zero, but at high fields, we observe the appearance of a voltage reminiscent of SdH oscillations. This voltage appears at the same field value where the measurements of Fig. 2(a) begin to deviate from each other. We emphasize that this nonlocal behavior is not generic to the quantum Hall regime, as Hall quantization generally occurs before its onset. Furthermore, it can be reduced without destroying the Hall quantization by increasing the temperature or the measuring current beyond certain limits.[2-4,6]

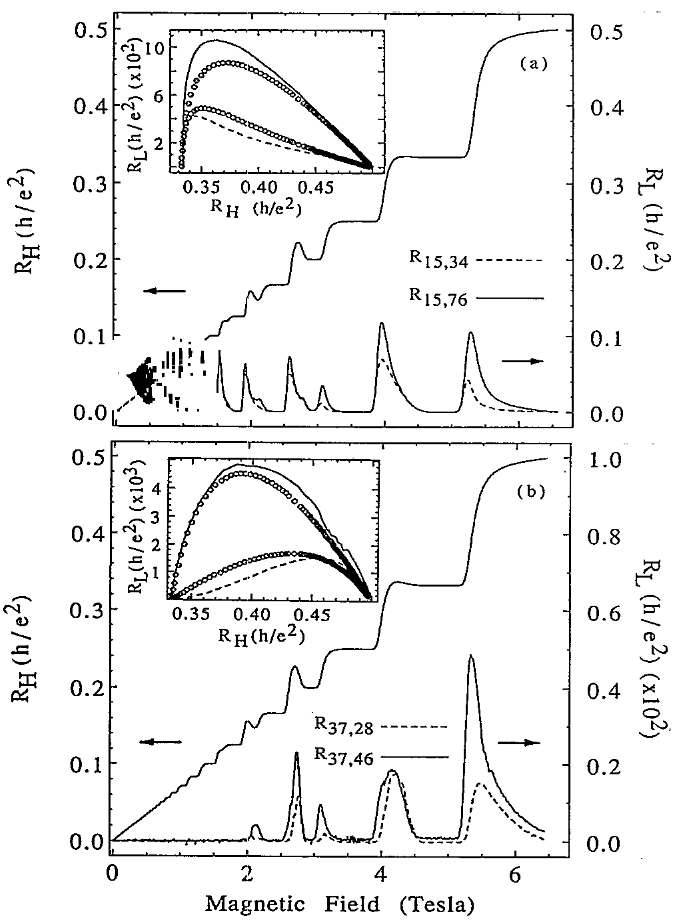

FIG. 2. (a) Longitudinal resistances $R_{15,76}$ (solid line) and $R_{15,34}$ (dashed line) and Hall resistance $R_{15,37}$. (b) Nonlocal longitudinal resistances $R_{37,46}$ (sold line) and $R_{37,28}$ (dashed line) and Hall resistance $R_{37,51}$. Insets: Experimental and theoretical (circles) plot of R_L vs R_H for the $N=3$ oscillation.

To understand this behavior, we begin with the edge-channel picture of the quantized Hall effect.[12-15] In this picture, only the transmission properties of the states at the Fermi level E_F need to be considered. These consist of $N-1$ states at the sample edges analogous to classical skipping orbits plus a topmost state that, in general, carries current in the bulk. In the quantized regime, all N states behave as edge channels and flow from one reservoir to the next in a direction determined by the magnetic field. Although the states have different velocities, the current carried by each channel is the same because of the well-known cancellation between the density of states and the velocity of an edge channel. The transport current flowing out of a reservoir j, which is at a chemical potential μ_j in excess of E_F, is $I_{\mathrm{out}} = N(e^2/h)\mu_j$, and the current flowing into the reservoir is the current that flows out of the preceding reservoir. The net current I_j is then

$$I_j = I_{\mathrm{out}} - I_{\mathrm{in}} = N(e^2/h)(\mu_j - \mu_{j-1}), \qquad (1)$$

where the probes are numbered in the same direction as the edge current flow. A voltage reservoir ($I_j = 0$) is thus at the potential of the reservoir feeding it, so $R_L = 0$. The voltage difference between voltage probes on opposite sides of the current source can be related to the net current by Eq. (1), giving $R_H = (1/N)h/e^2$.

Quantization is lost when E_F is near the center of the Nth Landau level, because extended states exist throughout the sample that can backscatter the Nth channel. If there is strong interchannel scattering, the remaining channels are also backscattered. However, recent experiments[5-7] have shown that in high-mobility samples there is strikingly little interchannel scattering between the Nth and the other channels. In the absence of sufficient interchannel scattering to establish local equilibrium (i.e., to keep current equally distributed among all the channels), it is not possible to describe the conductor by the usual resistivity[5,8-10] since in general the channels are at different local chemical potentials. If there is a finite amount of scattering present between edge and bulk channels (as is the case for most of the Landau levels in our experiments), a quantitative analysis is difficult. However, recent experiments by Alphenaar *et al.*[7] have shown that such scattering is negligible for the $N=3$ Landau level over ~ 0.1 mm (and is likely negligible over much longer distances). In other words, the edge channels continue to circulate around the periphery unaffected by the bulk.

In this case, we now show that a quantitative description is possible by treating the edge and bulk conducting pathways separately. We use a model of the conductor recently proposed by Szafer *et al.*,[11] where each segment j of the conductor is modeled by a barrier representing the backscattering of the topmost channel [Fig. 1(b)]. This barrier transmits only a portion t_j of the topmost (Nth) channel, but perfectly transmits the $N-1$ edge channels. The transmission probability t_j can be related to a resistivity by[14,16,17] $\rho_{xx}^N(L_j/W_j)=(h/e^2)(1-t_j)/t_j$, where L_j and W_j are the length and width of the jth segment and ρ_{xx}^N is the "resistivity" of the *Nth channel only*. All the t_j's are thus related to a single parameter ρ_{xx}^N by $t_j=1/[1+\rho_{xx}^N(L_j/W_j)]$. When ρ_{xx}^N is zero ($t_j=1$), the topmost channel behaves as an edge channel. As ρ_{xx}^N increases to infinity and the Nth channel is backscattered, the t_j's decrease to zero at a rate determined by their geometry, and R_H varies from $(1/N)h/e^2$ to $[1/(N-1)] \times h/e^2$ (with a concurrent peak in R_L).

To demonstrate some of the features of this model, we consider the probe, i.e., the reservoir plus barrier, shown in Fig. 1(b). Since the current in the topmost channel is partially backscattered at each barrier, the chemical potential in the topmost channel is now in general different from that in the other channels. The previous expression for the total current flowing out of probe j is modified to

$$I_j=(e^2/h)[(N-1)(\mu_j-\mu_{j-1})+\mu_j^N(\text{out})-\mu_j^N(\text{in})]\,, \tag{2}$$

where $\mu_j^N(\text{in})$ and $\mu_j^N(\text{out})$ are the chemical potentials of the Nth edge states that are going into and out of the jth probe. In addition, at each barrier j we have current-conservation equations of the form [see Fig. 1(b)]

$$\mu_j^N(\text{out})=\mu_j t_j+\mu_j^N(\text{in})(1-t_j)\,. \tag{3}$$

All the equations of the form (2) and (3) comprise a linear system of equations that can be solved for a given choice of current ($I_j=\pm I$) and voltage ($I_j=0$) probes, i.e., for a given resistance measurement. The probes are very important in this model, as we illustrate by considering the voltage $\mu_{j-1}-\mu_j$ measured between successive voltage reservoirs (a "longitudinal" voltage). From Eqs. (2) and (3) $\mu_{j-1}-\mu_j=[\mu_j-\mu_j^N(\text{in})]t_j/(N-1)$. The measured voltage depends upon t_j, the transmission property of a probe, and upon $\mu_j^N(\text{in})$, which depends upon the t's of the other probes.

We now compare the predictions of this model solved for the full eight-terminal device with the experimental results for the $N=3$ level. Within the model all measurable resistances of the conductor are a function of a single intensive ρ_{xx}^N that always varies from zero to infinity through the Nth transition region. Because of this, two resistance measurements can be plotted against one another as ρ_{xx}^N increases to infinity, and a curve is generated which depends only upon the geometry of the sample, and is independent of the detailed behavior of ρ_{xx}^N. Experimental data can be plotted in a similar fashion and compared to the theoretical curve with no parameters to fit. This is done in the insets of Fig. 2 for the $N=3$ Hall and longitudinal resistances. We see that the agreement is very good, both in terms of shape and amplitude. To experimentally test the generality of the R_L vs R_H curve, we consider the longitudinal and Hall measurements $R_L=R_{15,87}$ and $R_H=R_{15,37}$ at two different temperatures, shown in Figs. 3(a) and 3(b). R_L vs B and R_H vs B change with temperature, but R_L vs R_H is temperature independent, as seen in Fig. 4, and is in excellent agreement with theory.

The temperature dependence of R_L and R_H is due to the temperature dependence of ρ_{xx}^N. In Fig. 3(c), we show $\rho_{xx}^N(B,T)$ for the $N=3$ Landau level inferred from $R_{15,37}$ and from another Hall measurement, $R_{15,28}$

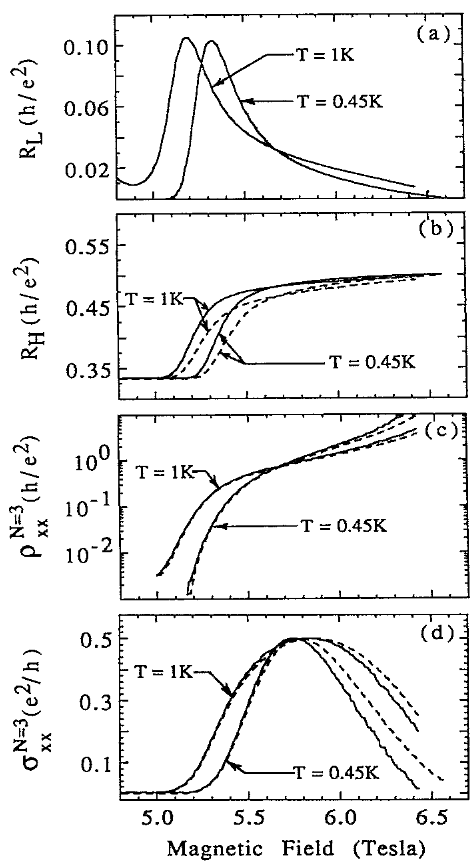

FIG. 3. (a) The longitudinal resistance $R_L=R_{15,87}$ at $T=0.45$ and 1 K. (b) The Hall resistances $R_H=R_{15,37}$ (solid line) and $R_{15,28}$ (dashed lne) at $T=0.45$ and 1 K. (c) The longitudinal resistivity ρ_{xx}^N of the $N=3$ Landau level derived from the Hall measurements. (d) The longitudinal conductivity σ_{xx}^N.

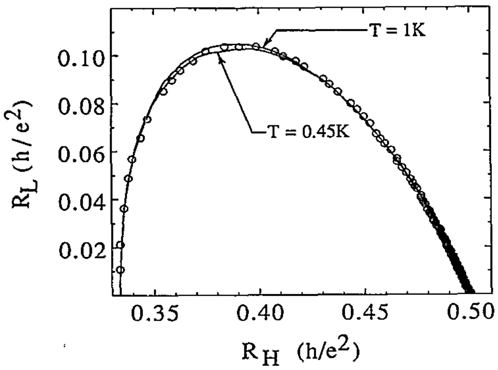

FIG. 4. Experimental (solid line) and theoretical (circles) plot of R_L vs R_H for $N=3$ at $T=0.45$ and 1 K, where $R_L = R_{15,87}$ and $R_H = R_{15,37}$ [Figs. 3(a) and 3(b)]. The curve is temperature independent because R_L and R_H are a function of only a single parameter, $\rho_{xx}^N(B,T)$.

(dashed lines). Although the shapes of the Hall risers are different [Fig. 3(b)] due to the differing geometry of the probes, we find that they yield the same values for ρ_{xx}^N, indicating that this quantity is indeed an intensive, i.e., geometry-independent, parameter. To understand its physical significance, we first convert ρ_{xx}^N to σ_{xx}^N, since σ_{xx}^N characterizes the dissipation associated with the bulk channel and is most directly related to the properties of the Landau level. To this effect, we use the theoretical[14,16] result that the Hall resistivity for a single channel is $\rho_{xy}^N = 1$. Inverting the resistivity tensor yields $\sigma_{xx}^N = \rho_{xx}^N/[1+(\rho_{xx}^N)^2]$, and using this equation, we obtain the results in Fig. 3(d). The maximum of σ_{xx}^N does not coincide with the maximum of the SdH oscillation or the midpoint of the Hall riser shown in Figs. 3(a) and 3(b). Rather, it roughly coincides with the place where curves at different temperatures cross, i.e., where the conducting properties are temperature independent. We believe that this represents the center of the Landau level, where conduction in the bulk exists, even at $T=0$. Furthermore, the asymmetry of the SdH oscillation in Fig. 3(a) does not reflect an asymmetry in σ_{xx}^N, but is instead related to the geometry of the sample, as can be demonstrated within the model.[11] Again, this points out that measured resistances are not related to the underlying conducting properties in a straightforward manner, and care must be taken in interpreting experimental results.

In conclusion, SdH oscillations and Hall risers are affected by the lack of equilibration between edge and bulk channels,[5-8] leading to nonlocal behavior in the measured resistances. When there is no interchannel scattering, the results can be quantitatively accounted for within a model where the bulk of the conductor is characterized by a single intensive parameter associated with the topmost Landau level only.

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