Pseudoreversibility Condition for Minimizing Radiation Loss at Discontinuities of Coupled Waveguides

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***Abstract*—A design condition is derived for lossless discontinuities of coupled waveguides by finding solutions, which satisfy time reversibility of guided modes. The derived condition is used as a bound for minimizing radiation loss. Even when there is strong coupling between guided modes, the radiation loss can be minimal. A comparison between different grating designs in a 2 × 2 backward coupler demonstrates that two coherently coupled gratings can significantly reduce radiation loss, as compared to a single grating.**

***Index Terms*—Integrated optics, mode coupling, optical waveguides, optimization methods, radiation loss.**

I. Introduction

GUIDED-TO-RADIATION mode conversion losses at optical waveguide discontinuities have been of concern since the dawn of integrated optics [1]. While many photonic devices depend in their operation on the interaction of more than one guided mode [2], most of the reports on minimizing radiation loss have focused on single mode devices, e.g., in [3]. Also, previous studies on multimode coupling require numerically intensive calculations to compare between different device designs, e.g., in [4].

In this letter, we consider radiation losses for coupled optical waveguides in which the discontinuity can scatter energy into other guided modes. By searching for solutions that most closely satisfy time reversibility of the guided modes, the guided-to-radiation mode conversion is minimized. Not only is this approach valid for weak perturbations, but also for strong intermode coupling. Example designs presented below demonstrate that the radiation loss can be quite small, even when intermode coupling is large.

II. Theoretical Background

A. Pseudoreversibility Condition

Consider the case of a slab waveguide discontinuity that supports M-guided system modes in Side 0 ($z < 0$) and an equal

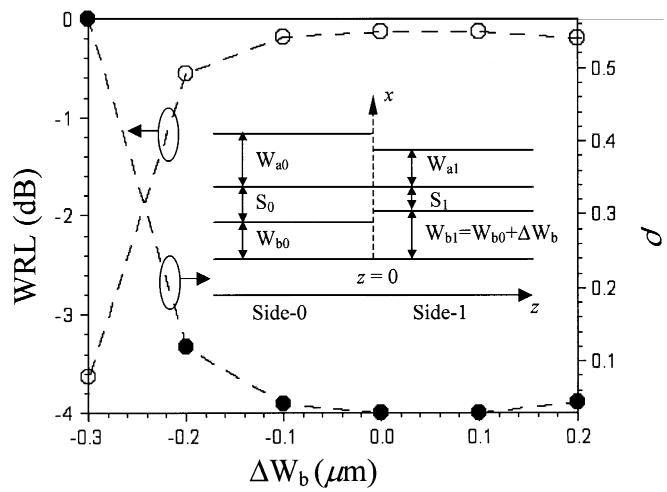

Fig. 1. WRL and ρ at $\lambda_o = 0.6328$ μm for the discontinuity (shown in side view) as a function of ΔW_b.

number of modes in Side 1 ($z > 0$). The amplitudes of incident, reflected, and transmitted guided modal fields, for this discontinuity, are related by the transmission and reflection matrices, T_{kl} and Γ_{kl}. The suffix kl describes the case where the guided modes are incident from Side k (0 or 1) to Side l (1 or 0). These matrices are obtained following the approach in [1]. It can be shown, using this approach, that the continuity of the transverse component of electric field at the discontinuity relates T_{kl} and Γ_{kl} by

$$T_{kl} = U_{kl} \left(I + \Gamma_{kl} \right) \qquad (1)$$

for both transverse electric (TE) and transverse magnetic (TM) modes. Here I is the identity matrix and U_{kl} is the matrix whose ij element, $u_{kl,\, ij}$, is the overlap integral between the transverse electric fields of the ith and jth guided modes across the junction. Unlike the TE case, the square of the refractive index profile appears in the overlap integral for TM modes. The sign of Γ_{kl} is positive because, in the derivation of (1), the transverse electric fields of the incident and reflected guided modes are added, as in [1]. Thus, any change in phase due to reflection of guided modes is included in Γ_{kl}. The inclusion of U_{kl} accounts for both guided-to-guided mode conversion directly, and also guided-to-radiation mode conversion loss through consideration of total power flow. In the case of no mode conversion, U_{kl} equals I, T_{kl} and Γ_{kl} become diagonal matrices, and (1)

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TABLE I
VALUES OF ρ AND WRL AT $\lambda_o = 0.6328$ μm FOR FOUR DISCONTINUITIES THAT HAVE NEARLY IDENTICAL AMOUNTS OF MODE COUPLING

W_{a1} (μm)	W_{b1} (μm)	$u_{01,00}$	$u_{01,01}$	$u_{01,10}$	$u_{01,11}$	ρ	WRL (dB)
0.6	0.6	0.929892	0.341878	-0.358154	0.925462	0.033558	-0.148242
0.4	0.37	0.92842	0.359161	-0.344045	0.926192	0.031574	-0.139331
0.5	0.48	0.931491	0.353300	-0.352615	0.932819	0.007997	-0.034870
0.3	0.26	0.923721	0.354666	-0.328394	0.891741	0.099093	-0.453191

reduces to M number of plane-wave-like equations[1] describing lossless scattering of the individual guided modes.

If guided modes are the only solution to the scattering problem in an isotropic power-conserving junction, then, according to the time-reversibility property of Maxwell's equations, they should satisfy the following conditions [6]:

$$T_{10}\,T_{01} + {\Gamma_{01}}^2 = I \tag{2a}$$

$$\Gamma_{10}\,T_{01} + T_{01}\,\Gamma_{01} = 0. \tag{2b}$$

The transmission and reflection matrices in (2) are all real, as is expected from normal incidence at an all-dielectric interface. Combining (1) and (2) gives the reversibility condition of guided modes

$$U_{10} = {U_{01}}^{-1}. \tag{3}$$

For the TE case $U_{10} = {U_{01}}^t$, which simplifies (3) to the matrix-orthogonality condition

$${U_{01}}^t = {U_{01}}^{-1}. \tag{4}$$

The remainder of the letter considers only TE modes.

B. Radiation Loss Minimization

The pseudoreversibility condition (4) represents a zero radiation-loss bound. The deviation from this bound can be characterized by the error matrix

$$\varepsilon = I - {U_{01}}^t U_{01}. \tag{5}$$

A scalar measure of this deviation is

$$\rho \equiv \max_{\|V\|=1} \|\varepsilon\, V\| \tag{6}$$

where V is an arbitrary vector and $\|\bullet\|$ denotes the Euclidean norm of that vector [7]. Since the parameter ρ is the largest amplification that the error matrix ε induces on a vector, it is a measure of worst-case (maximum) radiation loss (WRL). Since ε is symmetric, the value of ρ equals the magnitude of its largest eigenvalue [7]. To identify the minimum (actually, mini–max) radiation loss condition, for specific discontinuities, the value of ρ is minimized as a function of the achievable values of $u_{01,\,ij}$ (which depend on the junction-design parameters).

[1]For lossless scattering of a plane wave, which is incident normally from Side k to Side l of a dielectric interface, the continuity of electric field at the interface gives, $T_{kl} = 1 + \Gamma_{kl}$, where T_{kl} and Γ_{kl} are the scalar transmission and reflection coefficients [5].

III. EXAMPLE ANALYSES

The first example is used to demonstrate that the minimum value of WRL corresponds to the lowest value of ρ. The specific problem corresponds to finding the minimum WRL for the parallel slab waveguide structure in Fig. 1 (inset) as a function of the variations in W_{b1}, the width of the lower guide on Side 1. The widths of the other three guides are $W_{a0} = 0.6$ μm, $W_{b0} = W_{a1} = 0.4$ μm. The guides on Side 0 are separated by a gap $S_0 = 0.5$ μm. The gap on Side 1 decreases by an amount equal to the increase in the guide width W_{b1} such that $\Delta W_b \equiv W_{b1} - W_{b0} = \Delta S$ where $\Delta S \equiv S_0 - S_1$. The four guiding regions have identical core index of $n_g = 1.515$ and the gaps, cladding, and substrate have refractive index $n_c = 1.462$. The five-layer slab on Side 0 and Side 1 each support $M = 2$ guided TE system modes.

The computations involve calculating the local transverse fields distribution of the infinite structures on both sides of the junction. Then these results were used to calculate U_{01} and ρ. For some linear combination of incident-guided modes, radiation loss was determined, as in [1], from the difference in power between the incident and scattered guided modes. WRL was then found by varying the linear combination of the two incident modes until radiation loss was maximized. All these computations were carried out at a free-space wavelength, $\lambda_o = 0.6328$ μm. Fig. 1 shows that the minimum value of WRL takes place at the lowest value of ρ, which verifies the utility of using ρ as a minimization parameter.

A key point about (4) is its more general solution, $U_{01} \neq I$, which suggests that waveguide junctions with strong coupling between guided modes can have minimal guided-to-radiation mode conversion. For example, consider point (a) where $\Delta W_b = +0.2$ μm and point (b) where $\Delta W_b = -0.2$ μm in Fig. 1. These points have U_{01} matrices with the elements, $u^{(a)}_{01,\,00} = 0.775$, $u^{(a)}_{01,\,11} = -0.790$, $u^{(a)}_{01,\,01} = 0.600$, $u^{(a)}_{01,\,10} = 0.597$, $u^{(b)}_{01,\,00} = 0.989$, $u^{(b)}_{01,\,11} = 0.938$, $u^{(b)}_{01,\,01} = -0.003$, and $u^{(b)}_{01,\,10} = 0.006$. Although ${U_{01}}^{(b)}$ is closer to the identity matrix compared to ${U_{01}}^{(a)}$, the WRL of point (a) is ~0.36 dB less than that of point (b). Correspondingly, the value of $\rho^{(a)} = 0.044$ is less than $\rho^{(b)} = 0.119$.

Another important point in designing waveguide junctions is that they can have a relatively wide range of WRL while still having the same values of mode-conversion coefficients (off-diagonal U_{01} elements). Junctions similar in design to the previous example were evaluated at $\lambda_o = 0.6328$ μm. With $\Delta W_b = \Delta S$, both W_{b1} and W_{a1} are adjusted to make the off-

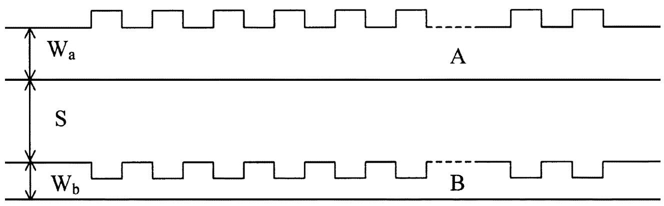

Fig. 2. Schematic of a grating-assisted backward coupler. The π phase shift between the two gratings is chosen to maximize the coupling coefficient for a pair of gratings of fixed depth.

diagonal matrix elements, $u_{01,01}$ and $u_{01,10}$, approximately identical (to within ±4%) among all four junctions in Table I. Despite nearly identical mode coupling, there is an order of magnitude change in WRL. The minimum value of WRL corresponds to a junction that most closely satisfies the matrix-orthogonality condition[2] in (4).

We note for this specific example that the minimum WRL happens to occur for $W_{a0} - W_{a1} \approx W_{b1} - W_{b0}$ or when the differences in width of guide A are about the same as in guide B. Similarly we suspect that a grating-assisted coupler, which is a periodic array of discontinuities, might have smaller radiation loss when there are small discontinuities on both guides rather than one larger (but equal reflection strength) discontinuity on one of the two guides. We look into this possibility in the selection of a 2×2 backward coupler for a pair of coupled waveguides for which $n_g = 1.562$, $n_c = 1.462$, $W_a = 0.4$ μm, $W_b = 0.2$ μm, and $S = 0.8$ μm as indicated in Fig. 2. Three different coupling structures (each designed to produce the identical coupling coefficient of 2.3 mm^{-1} at a resonance wavelength of 0.6302 μm) are evaluated in terms of radiation loss. Coupling structure G_{AB} (illustrated in Fig. 2) has grating depths on guides A and B of 30 and 20 nm, respectively. Also the grating is distributed coherently in parallel on both waveguides as shown in Fig. 2. For coupler G_A, the grating depth is 50 nm on guide A and 0 nm on guide B, while for coupler G_B, guide B has a grating depth of 60 nm and guide A has a depth of 0 nm.

The results of computations using a simulator that employs a bidirectional beam propagation method (BD-BPM) [8] are shown in Fig. 3. The minimum WRL is obtained using the two coherently coupled gratings G_{AB}. Of the three gratings, G_{AB} corresponds to the U_{01} for a single discontinuity that most closely approximates an orthogonal matrix, which is a similar observation as in the previous example. Note that the asymptotic value of WRL is approximately 0.7, 1.3, and 0.05 dB for G_A, G_B, and G_{AB}, respectively. The corresponding values of ρ are equal to 0.0043, 0.0106, and 0.0019, which is consistent with the simulation results.

It is interesting to note that coupler G_{AB} demonstrates extremely low radiation loss by producing very efficient coupling between the two supermodes of waveguides A and B. A practical use of such a low-loss structure would be as end reflectors for resonant cavities; e.g., in a chemical sensor. The low loss could greatly increase sensitivity through increase of the interaction time of the light with the chemical on the guide.

[2]For the 2×2 U_{01} matrices in Section III, the matrix-orthogonality condition gives, $u_{01,00} = \pm u_{01,11}$, $u_{01,01} = \mp u_{01,10}$, and $\Delta = \pm 1$, where Δ is the determinant of the matrix.

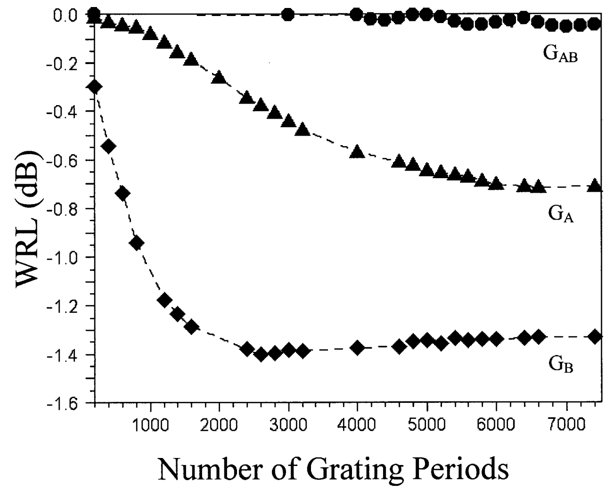

Fig. 3. WRL at resonance for three types of reflection gratings as calculated by BD-BPM. The resonance wavelength is 0.6302 μm.

IV. Conclusion

We have shown that it is possible to minimize radiation loss for discontinuities of coupled waveguides by searching for solutions that most closely satisfy time reversibility of guided modes. The search condition applies for both weak and strong perturbation on the guided modes. Minimization of the parameter ρ simplifies minimizing radiation loss over using repetitive numerical simulations of junction scattering. Example analyzes and designs were presented for the TE case only; however, the procedure is applicable to the TM case as well.

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